Precautionary Savings, Loss Aversion, and Risk: Theory and Evidence

WORKING PAPER 2303

JULY 2023

Sanjit Dhami

Narges Hajimoladarvish

Konstantinos Georgalos

Precautionary Savings, Loss Aversion, and Risk: Theory and Evidence

Sanjit Dhami* **Narges Hajimoladarvish †** **Konstantinos Georgalos‡**

July 13, 2023

Abstract

We consider a simple, two period, consumption-savings model with future income uncertainty that examines the interplay of savings, precautionary savings, loss aversion, and risk. We provide the relevant theory, followed by empirical tests based on subject-specific choices, and the measurement of subject-specific behavioral parameters such as loss aversion and present bias. We predict, and show empirically, that loss aversion reduces savings, and that those who are more loss averse are less likely to engage in precautionary savings. Present-bias reduces savings. We also show that decision makers save more in response to a mean preserving spread of future random incomes, and this response is strengthened by loss aversion. We term this as the loss aversion-hedging motive.

Keywords: Income uncertainty, precautionary savings; loss aversion; loss aversion-hedging.

JEL Classification: D01 (Microeconomic Behavior: Underlying Principles); D91 (Role and Effects of Psychological, Emotional, Social, and Cognitive Factors on Decision Making).

*Corresponding author. Professor of Economics, Department of Economics, Finance and Accounting, School of Business, Room 0.20, Teaching Centre, Brookfield, 266 London Road, University of Leicester, Leicester LE2 1RQ, UK. Phone: +44-116-2522086. Fax: +44-116-2522908. E-mail: sd106@le.ac.uk.

†Senior Research Fellow, Centre for Social and Behaviour Change, Ashoka University, Delhi Office No. 222, Second Floor, Okhla Industrial Estate - Phase III, New Delhi, 110020, India. E-mail: narges.hajimoladarvish@ashoka.edu.in.

‡Senior Lecturer, Lancaster University Management School, Department of Economics, United Kingdom, LA1 4YX E-mail: k.georgalos@lancaster.ac.uk.

1 Introduction

Consider a two period, consumption-savings problem with future income uncertainty, and a single good, but no financial assets. Suppose that first period income is non-stochastic. However, second period income is stochastic; it can either be low (bad state) or high (good state). The key insight from macroeconomic models is that under such conditions, individuals might save for two main reasons. First, in order to smooth consumption over time. Second, and this is our main focus, to hedge against the uncertainty caused by stochastic future income.

Different motivations arise for coping with future income uncertainty and we may, purely for pedagogical reasons, distinguish between the micro approach and the macro approach.

1. In microeconomics and in behavioral economics, the two key motivations are *risk aversion* and *loss aversion*. Yet, an increase in risk aversion in such a model may not always lead to unambiguous predictions about savings.[1] Loss aversion is a central concept in behavioral economics and underpins some of the most successful applications of behavioral economics in humans and animals.[2] The literature has also attempted to derive a link between loss aversion and savings (Aizenman, 1998; Bowman et al., 1999; Siegmann, 2002; Koszegi and Rabin, 2009; Park, 2016; Pagel, 2017; Ibanez and Schneider, 2023); we note our differences from this literature below.

2. In macroeconomics, the focus has been on identifying the *precautionary savings motive* for hedging against future income uncertainty.[3] Leland (1968) first showed in a two period model that if the third derivative of the utility function is strictly positive ($u''' > 0$), the decision maker engages in *precautionary savings*. This takes the form of increasing current savings at the expense of current consumption, hence, leading to higher consumption growth.[4]

We give the benchmark results under classical preferences, but our main interest is in prospect theory preferences of the Köszegi-Rabin form (Köszegi and Rabin, 2006).[5] In our two period

[1]The reason is that changes in risk aversion not only alter the risk hedging motive, but also alter the marginal utility of income (see Proposition Proposition 2(iii) below). However, under certain conditions, risk aversion may lead to an increase in savings in the presence of loss aversion (Bowman et al., 1999).

[2]Loss aversion implies that losses are more aversive than the satisfaction derived from gains of the same magnitude. The mean and the median values of loss aversion typically cluster around a value of 2; see below. For surveys of the rich applications of loss aversion, see Kahneman and Tversky (2000) and Dhami (2019, Vol. 1). Indeed, associating risk aversion with the shape of the utility function alone, as is the case under expected utility theory, is empirically refuted on account of non-linear probability weighting (Kahneman and Tversky, 2000; Starmer, 2000). There is also a claim that risk aversion has little empirical power to explain cautionary behavior once loss aversion is fully taken into account (Novemsky and Kahneman, 2005).

[3]In models with a richer menu of assets than ours, hedging against future income uncertainty can take many other forms. This includes, but is not restricted to, investing in assets with correlated returns, and taking either a short or a long position in substitute securities. We abstract from these considerations in our model that is designed to be tested stringently in lab experiments.

[4]This result was extended to multiple periods (Sibley, 1975; Miller, 1976). Other extension included the implications for savings from different sources of income (Sandmo, 1970; Skinner, 1988); relation between risk preferences and precautionary savings (Dreze and Modigliani, 1972; Kimball, 1990); and the computation of numerical solutions (Zeldes, 1989). There are several applications of these ideas; see Carroll and Kimball (2008) and Lugilde et al. (2019) for surveys. The classical precautionary savings model has also been adapted in several other directions such as financial inclusion, subjective expectations, migrant networks, on the job search, illiquid assets and portfolio choice (Giles et al., 2007; Lise, 2013; Deidda, 2013; Bayer et al., 2019; Christelis et al., 2020) but this is not the focus of our work.

[5]Despite some anomalies, it is widely recognized that prospect theory is the best available decision theory 'jointly'

model, decision makers receive an initial endowment of a single good at time $t = 1$ and must decide between current consumption and savings. At time $t = 2$, the income of the decision maker comprises of savings carried over from time $t = 1$, plus a random income component. Random income can take two possible values– a negative value (negative shock) with probability $0 < p < 1$, and a positive value (positive shock) with probability $1 - p$. At time $t = 1$, the decision maker only knows the distribution of the shocks. Thus, at time $t = 2$, in the event of a negative shock to income, the decision maker may be in the *domain of losses,* relative to the reference point, where loss aversion bites.

We show, theoretically, that for 'loss averse' decision makers, the classical result on the sufficiency of $u''' > 0$ for precautionary savings still holds, but it does not hold for 'loss tolerant' subjects. In the Appendix, we show that, under plausible conditions, loss averse decision makers may exhibit precautionary savings even when $u''' = 0$. However, while these results are, we believe, new, they are purely of theoretical interest, partly because the third derivative of the utility function is unobserved. By contrast, our focus in this paper is to provide testable predictions on human behavior, and then to stringently test them. We now turn to these predictions.

We are largely interested in the effect of loss aversion on savings, which has two opposing temporal effects in the model. Loss aversion applies at both dates, $t = 1$ and $t = 2$. At time $t = 1$, reducing current consumption to increase savings by a unit is aversive to loss averse subjects. At time $t = 2$, losses only occur with probability $0 < p < 1$ in the event of a negative shock to income; and only in this state does loss aversion bite. Hence, the time $t = 2$ loss aversion offsets time $t = 1$ loss aversion, in marginal utility terms, by a diminished amount due to $0 < p < 1$. Thus, on net, the time $t = 1$ effect dominates, and an increase in loss aversion is predicted to reduce current savings (Proposition 2(i)). Furthermore, we show that loss averse subjects are less likely to engage in precautionary savings relative to loss tolerant subjects (Corollary 1) and this provides us with an important method of directly testing our predictions.

By observing the optimal saving choices of subjects, we are able to compute the gap between optimal consumption at time $t = 1$, c_1^*, and expected second period consumption at time $t = 2$, Ec_2^*. Under the classical certainty equivalence result, that arises under quadratic utility, we have $c_1^* = Ec_2^*$. The precautionary savings motive gives an additional inducement to save, so that $c_1^* < Ec_2^*$; and we refer to the flip side, $c_1^* > Ec_2^*$, as 'reckless undersaving.' This allows us to classify subjects into these three categories.

Risk aversion is predicted to have an ambiguous effect on savings. It is important to note that the predictions of our model hold for 'each' decision maker. This requires us to estimate subject-specific loss aversion in order to conduct a stringent test of our theory. Testing the classical prediction that relates precautionary savings to the sign of the third derivative of the utility function would require determining the sign of this derivative for each subject; an exercise we do not engage in.[6]

under risk, uncertainty, and ambiguity. For book length treatments of this assertion, see Wakker (2010) and Dhami (2019, Vol. 1).

[6]No such direct tests, based on the third derivative of the utility function, have been conducted in the literature. Several indirect tests based on approximations of the Euler equation, regression analysis, and structural models based on micro data have been conducted (Guiso et al., 1992; Carol and Samwick, 1998; Lusardi, 1998; Engen and Gruber, 2001; Gourinchas and Parker, 2002; Cagetti, 2003). For a survey, see Carroll and Kimball (2008). There

1.1 Experiments

We conduct a suitably incentivized lab experiment with 79 students from at an Experimental Economics Lab in the UK in March 2023. Our empirical tests are theory driven, direct, stringent, and based on subject-specific behavioral data. Subjects in our experiments engage in two different tasks.

1. In the *consumption-savings task*, we confront subjects with our two period model with second period income uncertainty. The model presented in the experiments is identical to the model used to derive our theoretical predictions.

2. In the *lottery choice task*, subjects make choices between risky lotteries that allows us to estimate the parameter of loss aversion for each subject in our experiment. Our method has similarities with the methods in Abdellaoui (2000) and Gächter et al. (2022).

1.2 Findings

The mean subject-specific loss aversion parameter in our experiment is 1.6571 and the median value is 1.6609.[7] On average, 46% of the savings choices in our experiment are consistent with the precautionary saving motive, 18% with the classic certainty equivalence result, and 36% with reckless undersaving.

Our empirical findings largely confirm our theoretical model. An increase in loss aversion decreases individual savings. Loss aversion, relative to loss tolerance, decreases the odds of precautionary saving by 52%, on average. Being loss averse, relative to loss tolerant, decreases the probability of precautionary saving behavior by almost 20%. This demonstrates a link between the micro and macro motives that was mentioned above.

We are also able to classify individuals into those who are present-biased in the sense of exhibiting preferences reversals. Present-biased individuals are less likely to engage in precautionary saving behavior, as one would expect. The statistical effects of present bias and loss aversion are comparable, and both effects are significant at the 1% level. This result speaks to the growing literature on risk and time preferences; for a survey, see Dhami (2019, Vol. 3). Being male, relative to female, decreases the odds of precautionary saving behavior by 39%. This result speaks to the large literature on the greater risk-seeking and overconfidence among men, relative to women; for a literature survey, see Dhami (2016).

Older individuals and those who spend more time deliberating on saving decisions are more likely to engage in precautionary saving. Keeping fixed the expected value of the shock at time $t = 2$, we show that precautionary savings are more likely to arise when the magnitude of the negative shock in the bad state is higher.

Finally, and in an important result, we show that a mean preserving spread of the random time $t = 2$ income induces decision makers to save more. However, unlike the classical explanation

are concerns about each of the methods employed in terms of the empirical proxies used, truly exogenous variation in uncertainty, and the ability to control for potential confounds.

[7] In their meta study, Brown et al. (2023) find that the mean loss aversion coefficient is 1.955. Gächter et al. (2022) find that the mean subject-specific loss aversion for riskless choice is 2.12 and the median is 1.73; for risky choice, median loss aversion is 1.33 and the loss aversion parameters for risky and riskless choice are correlated.

based on risk-aversion, we show that this is on account of the loss aversion motive. Even when the shocks to random income are symmetric, loss averse decision makers perceive the downside risk to be higher than it actually is, and save more. We term this the *loss averse-hedging motive*, unlike risk-hedging. For instance, the most extreme shock (gain-lose 2000 units with equal probability) relative to our reference shock (gain-lose 500 units with equal probability) induces a loss averse decision maker to save an extra 481 units relative to a loss tolerant decision maker; and this difference is statistically significant. We show that it is difficult to reconcile this empirical finding with the main theoretical models in the literature.

1.3 Related Literature

In our model the decision maker experiences loss aversion from sacrificing current consumption at time $t = 1$. This channel was also used by Thaler and Benartzi (2004) as the 'basis' of their influential SMart savings plan. As described above, this led to our prediction (which we successfully tested) that loss aversion reduces savings. However, several papers with a dynamic structure ignore the effect of loss aversion at time $t = 1$ and take account of only the effect of loss aversion in mitigating the effect of the bad income state in the future at time $t = 2$ (Aizenman, 1998; Siegmann, 2002; Koszegi and Rabin, 2009; Park, 2016; Pagel, 2017; Ibanez and Schneider, 2023). By ignoring the effect of loss aversion on current consumption at time $t = 1$, these models predict that loss aversion will increase savings, the opposite of our prediction, and one that is inconsistent with our empirical result.

Of the papers cited in this subsection, only Ibanez and Schneider (2023) provide an empirical test supporting their prediction using observational savings data from low income individuals in Bogotá, Colombia. However, their use of observational data poses challenges in measuring the relevant variables, and there are important differences in their paper from ours.[8] Their main result that 'loss aversion increases savings' is inconsistent with our experimental data, which shows a highly statistically significant and 'negative effect of loss aversion on savings.'

1.4 Schematic outline

The schematic outline of the paper is as follows. Section 2 describes the dynamic model. Section 3 gives several benchmark results, including the classical results on precautionary savings. Section 4 gives the main testable predictions of our model for prospect theory preferences. Section 5 describes our experiments, procedures and our data. Section 6 gives the regression results. Section

[8]Ibanez and Schneider (2023) take the existing value of the stock of an individual's monetary assets as a measure of savings, made over several periods. In the real world, future income uncertainty might be non-stationary. Yet, the predictions of their model are for savings made in response to 'a specific' stationary probability distribution of uncertain future incomes. They measure income risk from the self-reported risk from becoming unemployed. An advantage of our lab experiments is that we can tightly control for savings and risk to correspond exactly to the sense used in our theoretical model. Furthermore, their theoretical results rely on a different equilibrium concept (PPE in Koszegi and Rabin, 2009) and hold only for a Taylor's series approximation around a shock value of zero. By contrast, our empirical results are for symmetric risk shocks where high magnitudes of the shocks (e.g., $-500, -1000, -2000$ units) rule out any approximations around a shock value of 0. They identify precautionary savings with a positive third derivative of the utility function, which is unobserved. By contrast, and as discussed above, we use the fundamental result on the sign of $c_1^* - Ec_2^*$ to classify subjects as precautionary savers. Hence, they do not classify subjects into those who undertake precautionary savings, or undertake reckless undersavings. Their measurement of loss aversion also uses a different method to ours.

7 gives the conclusions. All proofs are relegated to the Appendix, which also contains additional results to show that, under plausible conditions, a positive third derivative is not necessary for precautionary savings even for loss averse individuals.

2 The model

Consider a decision maker who lives for two time periods, where time $t = 1, 2$; and has initial non-stochastic income, $y > 0$, at time $t = 1$, and stochastic income z at time $t = 2$ (specified below). Income y can be either consumed, c_1, or saved, $s \geq 0$. All income at time $t = 2$ is fully consumed, c_2. We assume that the interest rate on savings equals zero.[9] Thus, the budget constraints at times $t = 1$ and $t = 2$ are given, respectively, by

$$c_1 + s = y, \tag{2.1}$$

$$c_2 = s + z. \tag{2.2}$$

The stochastic income, z, at time $t = 2$, takes two possible values.

1. In the *bad state,* which occurs with probability $p \in (0, 1)$, $z = \varepsilon_l < 0$.

2. In the *good state*, which occurs with probability $1 - p$, $z = \varepsilon_h > 0$.

Thus, in the bad state, the lowest possible value of c_2 is ε_l (if $s = 0$) and in the good state, the highest possible value of c_2 is $y + \varepsilon_h$ (if $s = y$). Hence, the set of all possible outcomes for c_2 is given by the compact set $X = [\varepsilon_l, y + \varepsilon_h]$. The expected value of the random second period income is

$$Ez \equiv \overline{z} = p\varepsilon_l + (1 - p)\,\varepsilon_h, \tag{2.3}$$

where E is the expectation operator that captures uncertainty with respect to the realization of the random income z.

In each of the two time periods, the decision maker has a instantaneous utility function that is of the Köszegi-Rabin form.[10] One needs to first define the relevant *reference points.* Let ω_1 and ω_2 be, respectively, the reference points for consumption at times $t = 1$ and $t = 2$; we specify reasonable bounds on the reference points below. Hence, the decision maker maximizes the following, undiscounted, two period utility function[11]

$$U = v(c_1; \omega_1) + E\left[v(c_2; \omega_2)\right]; \; c_1 \in [0, y]\,, c_2 \in X, \tag{2.4}$$

[9]Alternatively, in the analysis of precautionary savings, we could have assumed a positive interest rate that equaled the discount rate so that they both cancel out in the relevant Euler equation (Blanchard and Fischer, 1989). But this requires assuming a steady state, which might be an unreasonable assumption in a one shot experiment. It is more persuasive to make the assumption of zero interest rate that can be easily implemented in experiments.

[10]For an introduction to prospect theory and Köszegi-Rabin preferences, and their applications, see Köszegi and Rabin (2006, 2009) and Dhami (2019, Vol. 1).

[11]The absence of discounting is required to ensure that the discount rate equals the zero interest rate, which, as noted above, is critical to define precautionary savings. Our results go through with a positive discount rate and a positive interest rate if we make the steady state assumption that they are identical. The time gap between the two time periods in our experiment is 1 month, hence, the discount rate over this interval is likely to be reasonably small. Furthermore, we demonstrate the robustness of our measurement methods to considerations of a positive discount rate below.

where

$$v(c_t;\omega_t) = u(c_t) + \mu g\left(c_t;\omega_t\right);\ \mu \geq 0,\ t = 1, 2. \tag{2.5}$$

In (2.5), the utility function from the 'absolute level' of consumption at time $t = 1, 2$, $u : \Re \to \Re$, is twice continuously differentiable, strictly increasing, and if $c_t \geq 0$, for $t = 1, 2$, it is also strictly concave ($u' > 0$, $u'' < 0$).[12]

In Köszegi-Rabin preferences, the second component in (2.5) is known as *gain-loss utility*. Gain-loss utility, g, at time $t = 1, 2$ depends on the value of consumption, c_t, relative to the reference point, ω_t, and $\mu \geq 0$ is the relative weight on gain-loss utility. Gain-loss utility, which applies under certainty and under risk, is given as in prospect theory[13]

$$g\left(c_t, \omega_t\right) = \begin{cases} \left(c_t - \omega_t\right) & \text{if} \quad c_t \geq \omega_t \\ -\lambda\left(\omega_t - c_t\right) & \text{if} \quad c_t < \omega_t \end{cases}. \tag{2.6}$$

In (2.6), the parameter λ is known as the parameter of loss aversion and this requires that $\lambda > 1$. In other words, losses are more aversive than equivalent gains, and this has massively expanded the explanatory power of economic theory.[14] However, recent empirical work, using experiments, has also found the presence of a significant number of loss-tolerant subjects (Chapman et al., 2022; Dhami et al., 2022); this corresponds to the case $\lambda < 1$. We allow for both cases in our empirical analysis but we continue to refer to λ as the parameter of loss aversion.

The classical case, without reference dependent preferences, is recovered as a special case of this model when we set the parameter of gain-loss utility $\mu = 0$.

There is extensive evidence that under certainty, the status-quo provides a good reference point (Kahneman and Tversky, 2000; Dhami, 2019, Vol. 1). For this reason, we take the time $t = 1$ reference point, ω_1, as the status-quo income, thus

$$\omega_1 = y. \tag{2.7}$$

It is less clear how reference points are formed under risk and uncertainty. Proposals include rational expectations, the expected value of the future income, or a fraction (greater than or less than 1) of the expected value; for a survey, see Dhami (2019, Vol. 1). In order to enhance the generality of our results we do not specify an exact reference point, but we specify the following eminently plausible bounds on ω_2 and thereby demonstrate the robustness of our results to a range of reference points.

We believe that it is not plausible to assume that ω_2, exceeds the highest possible time $t = 2$ income, which arises in the good state when $c_2 = s + \varepsilon_h$.[15] Hence, we assume that

$$s + \varepsilon_h > \omega_2, \forall s, \tag{2.8}$$

so in the good state, the individual is always in the domain of gains. Analogously, we assume that in the bad state of the world, the decision maker is always in the domain of losses

$$s + \varepsilon_l < \omega_2, \forall s. \tag{2.9}$$

[12] We consider the implications of $c_t < 0$ for the shape of utility function in the Appendix.

[13] In classical prospect theory preferences, we have $v(c_t;\omega_t) = g\left(c_t;\omega_t\right);\ \ t = 1, 2$.

[14] For extensive applications, see Kahneman and Tversky (2000), and Dhami (2019, Vol. 1).

[15] Note that at time $t = 2$, when forming the reference point, ω_2, the decision maker already knows the first period savings, s. Neither the status-quo interpretation of reference points, nor simple expected values, nor rational expectations of the reference point would allow for the contrary assumption.

Example 1 *Suppose, for instance, the second period reference point equals an arbitrarily weighted average of second period income in the different states, i.e.,* $\omega_2 = s + [w_l \varepsilon_l + w_h \varepsilon_h]$*, where* $w_l, w_h > 0$ *and* $w_l + w_h = 1$*; and* w_l, w_h *are the respective weights on the two shocks. A special case arises when* $w_l = p$*,* $w_h = 1 - p$ *so that* $\omega_2 = s + \overline{z}$*, where* $\overline{z} = p\varepsilon_l + (1-p)\varepsilon_h$*; in this case, the reference point is simply the expected value of income. For the arbitrarily weighted case,* $\forall s$ *we have that second period consumption* $c_2 = s + z$*:* $s + \varepsilon_l < \omega_2$ *and* $s + \varepsilon_h > \omega_2$*, in accordance with (2.8), (2.9). Thus, the bounds in (2.8), (2.9) accommodate a very large range of possible human behaviors in forming reference points. Our results are robust to any such behavior.*

3 General solution and some benchmark results

Substituting the budget constraints from (2.1) and (2.2) into the objective function in (2.4), and using (2.8), (2.9), the unconstrained optimization problem of the decision maker is

$$s^* \in argmax\ U = [u(y-s) + \mu g(y-s;\omega_1)] + [Eu(s+z) + \mu E g(s+z;\omega_2)],\ s \in [0,y], \quad (3.1)$$

where

$$g(y-s;\omega_1) = -\lambda s, \quad (3.2)$$

$$Eu(c_2) \equiv Eu(s+z) = pu(s+\varepsilon_l) + (1-p)\,u(s+\varepsilon_h), \quad (3.3)$$

$$Eg(s+z;\omega_2) = p\lambda(s+\varepsilon_l-\omega_2) + (1-p)(s+\varepsilon_h-\omega_2). \quad (3.4)$$

We now explain each of the expressions in (3.1)–(3.4). In (3.1), intertemporal Köszegi-Rabin utility is the sum of the utilities in each time period. Using (2.1) and (2.7), we have $c_1 - \omega_1 = y - s - y = -s < 0$. Thus, using the second row in (2.6), the time $t = 1$ gain-loss utility is $g(y-s,\omega_1) = -\lambda s$. The expected value of time $t = 2$ consumption, $Eu(c_2)$, in (3.3), takes expectations over the realizations of the random income, z, over the two states of the world. Finally, in the determination of second period gain-loss utility, $Eg(s+z,\omega_2)$, we take account of the relation between c_2 and ω_2 specified in (2.2), (2.8) and (2.9). With probability p, the bad state occurs and $s + \varepsilon_l < \omega_2$ so the second row of (2.6) applies; and with probability $1-p$, the good state occurs and $s + \varepsilon_h > \omega_2$, so the first row of (2.6) applies.

Differentiating (3.1) with respect to s, we get

$$\frac{\partial U}{\partial s} = -u'(y-s) - \mu\lambda + E\left[u'(s+z)\right] + \mu\left(p\lambda + (1-p)\right). \quad (3.5)$$

The second order condition is

$$\frac{\partial^2 U}{\partial s^2} = u''(y-s) + E\left[u''(s+z)\right] < 0.$$

Since the objective function is strictly concave and defined over a compact set, $s \in [0,y]$, there is a unique solution. At an interior solution, we have

$$\frac{\partial U}{\partial s} = -u'(y-s^*) - \mu\lambda + E\left[u'(s^*+z)\right] + \mu\left(p\lambda + (1-p)\right) = 0 \quad (3.6)$$

3.1 The classical benchmark certainty equivalent result ($\mu = 0$)

The classical certainty equivalent result relies on two assumptions. First, the utility function is quadratic. Second, there is no gain-loss utility, so $\mu = 0$. Thus, the utility function is given by

$$u(c_t) = c_t - \frac{a}{2}c_t^2,\ a > 0,\ c \geq 0,\ t = 1, 2. \tag{3.7}$$

Using (2.2) and (3.7), $Eu'(c_2) = p(1 - a(s + \varepsilon_l)) + (1 - p)(1 - a(s + \varepsilon_h))$, or

$$Eu'(c_2) = 1 - as - a\overline{z} = u'(Ec_2), \tag{3.8}$$

where $\overline{z}$ is defined in (2.3).[16]

Using the Euler equation (3.6), and the intermediate result in (3.8), the budget constraints, (2.1), (2.2), and restricting $\mu = 0$, at an interior solution, we have

$$u'(c_1) = Eu'(c_2) = u'(Ec_2).$$

$$\Rightarrow c_1 = Ec_2. \tag{3.9}$$

We can also solve out for the optimal level of savings. We have $Ec_2 = s + \overline{z}$, so using (2.1), we can rewrite (3.9) as $y - s = s + \overline{z}$, which can be solved out for the optimal savings level, superscripted with two stars to distinguish this special case,

$$s^{**} = \frac{y - \overline{z}}{2}. \tag{3.10}$$

Optimal savings are positive if $y \geq \overline{z}$, and negative if $y < \overline{z}$. The level of savings s^{**} comes about without any consideration of precautionary savings, purely to smooth income across the two time periods. In the special case when $\overline{z} = 0$, we have $s^{**} = \frac{y}{2}$, so that the decision maker smoothes income equally between the two time periods.

This gives rise to the *certainty equivalent result* because the optimal solution to savings in the following two cases is identical.

1. The decision maker receives at time $t = 2$, the non-stochastic income $s + \overline{z}$ with certainty.
2. The decision maker receives, as in our model, at time $t = 2$, the stochastic income z, in addition to first period savings, s, so that the second period income, $s + z$, is random.

The literature uses departures from the condition, $c_1 = Ec_2$, stated in (3.9), to identify *precautionary savings*, and its flip side, *reckless undersaving*. We summarize the standard result in the next definition.

Definition 1 *Let s^{**} be given in (3.10).*
(i) If $c_1 < Ec_2$ ($\Leftrightarrow s > s^{**}$) then the individual engages in precautionary savings.
(ii) If $c_1 > Ec_2$ ($\Leftrightarrow s < s^{**}$) then the individual engages in reckless undersaving.

Thus, the precautionary savings motive induces the decision maker to save an amount even greater than s^{**}, over and above what is required to perform the income smoothing role; while reckless undersavings induces them to save less. The next subsection clarifies why these cases might arise.

[16] In (3.8), we have used the fact that $u'(c_2) = 1 - ac_2$ and $Ec_2 = s + \overline{z}$.

3.2 Precautionary savings in the classical model ($\mu = 0$)

We show below in Proposition 1 the conditions under which precautionary savings arise in the classical model in the absence of gain-loss utility, $\mu = 0$.

Proposition 1 *Suppose that $\mu = 0$.*
(a) If for all $x \in X$ we have $u'''(x) > 0$, then the decision maker engages in precautionary savings ($c_1 < Ec_2$).
(b) If, for all $x \in X$ we have $u'''(x) < 0$, then the decision maker engages in reckless undersaving ($c_1 > Ec_2$).

From Proposition 1, when $u''(x) < 0$ and $u'''(x) > 0$, the decision maker reduces first period consumption relative to expected second period consumption. This gives rise to precautionary savings.

4 Optimal savings in the Köszegi-Rabin model, $\mu > 0$

4.1 The effect of loss aversion on savings

The comparative static effects of loss aversion on savings in the Köszegi-Rabin model are stated in the next proposition, followed by a discussion.

Proposition 2 *Consider the optimization problem with Köszegi-Rabin preferences in (3.1) and suppose that $\mu > 0$.*
(i) Optimal savings are decreasing in the magnitude of loss aversion, λ.
(ii) If the decision maker is loss averse ($\lambda > 1$), then optimal savings are 'decreasing' in the magnitude of the gain-loss parameter, μ. However, for loss tolerant decision makers ($\lambda < 1$) optimal savings are 'increasing' in μ.
(iii) For the CRRA utility function, $u(c) = \frac{1}{1-\gamma}c^{1-\gamma}; \gamma > 0, \gamma \neq 1$, the comparative static effect of the parameter of risk aversion, γ, on optimal savings, is ambiguous.

Discussion of Proposition 2: From the first order condition (3.6), loss aversion has two opposing intertemporal effects. Suppose that time $t = 1$ savings increase by 1 unit. This results in a time $t = 1$ loss in utility of $\mu\lambda$ (second term on the RHS of (3.6)). However, in the future, at time $t = 2$, losses only occur with probability $0 < p < 1$ and it is only in this state of the world that loss aversion bites. Hence, a unit increase in current savings offsets future loss aversion, in marginal utility terms, by $p\mu\lambda < \mu\lambda$. Thus, on net, an increase in loss aversion reduces current savings (Proposition 2(i)).[17] This is the main prediction of our model that we test with our data and it leads to a testable prediction for precautionary savings that we give in Corollary 1 below.

From the first order condition (3.6), a unit increase in the parameter of gain-loss utility, μ, leads to two effects. It increases current loss in utility from an extra unit of savings by λ. But it also leads to an increase in future marginal utility, from the extra unit of savings, equal to $p\lambda + (1 - p)$.

[17] This effect would be even stronger if future utilities were discounted. If the discount factor were $0 < \delta < 1$, then $\delta p\mu\lambda < p\mu\lambda < \mu\lambda$.

The net effect is a change in marginal utility by the term $(1-p)(1-\lambda)$ which is positive if $\lambda < 1$ (loss tolerant) and negative if $\lambda > 1$ (loss averse); this is the content of Proposition 2(ii).

Recall that we allow for, and measure, heterogeneity in the individual-specific parameter of loss aversion. Furthermore, our result does not depend on a particular specification of individual reference point, except for the conditions in (2.8), (2.9) that are fairly general (Example 1). By contrast, the effect of risk aversion on optimal savings is ambiguous (Proposition 2(iii)), and this results is well known in the literature.

From Proposition 2(i), the optimal savings of loss averse subjects ($\lambda > 1$) are predicted to be lower than the optimal savings of loss tolerant subjects ($\lambda < 1$). Hence, one potential testable implication of this result is that loss averse decision makers are less likely to engage in precautionary savings, relative to loss tolerant decision makers (see Corollary 1 below). However, the prediction in Proposition 2(i) can be directly tested, and we do so in our empirical exercise.

In testing the predictions, we do not observe the sign of the third derivative of the utility function, u''', for individual subjects. From Proposition 2(iii), we know that risk aversion has ambiguous effects on precautionary savings. Hence, even if we could accurately measure attitudes to risk, it would not necessarily be a test of the model. Thus, one needs to focus on testing other implications of the model.

We can, for each subject, observe their optimal savings, s^*. This allows us to compute their optimal first period consumption c_1^*, and their expected second period consumption Ec_2^*. Using Definition 1, we can then test if $c_1^* = Ec_2^*$ (certainty equivalence), $c_1^* < Ec_2^*$ (precautionary savings), or $c_1^* > Ec_2^*$ (reckless undersaving).

4.2 The effect of loss aversion on precautionary savings

Recall that the precautionary savings result requires us to explicitly assume that the interest rate and the rate of time preference are identical. In our experiments, the interest rate is zero. However, subjects in our experiments might have a strictly positive rate of time preference. Indeed, for about 28% of our subjects, we find the phenomena of *preference reversals*. One can show that a direct implication of Proposition 2(i) is that precautionary savings are more likely to arise if the parameter of loss aversion is lower. Or that more loss averse subjects are less likely to engage in precautionary savings. This result holds in the presence, but also in the absence, of time discounting, as we now demonstrate.

Corollary 1 *Precautionary savings are more likely to arise if the parameter of loss aversion is lower. In particular, loss averse subjects ($\lambda > 1$) are less likely to engage in precautionary savings as compared to loss tolerant ($\lambda < 1$) subjects. This result can be demonstrated in the presence, but also in the absence, of time discounting.*

We use Corollary 1 to test the predictions of our model on precautionary savings.

4.3 Optimal savings and means preserving spreads of income

In our experiments, we study the effects of mean preserving spreads of the random time $t = 2$ income, z. This gives rise to two questions. First, what is the predicted effect on optimal savings when a mean preserving spread of incomes takes place. Second, how does loss aversion influence the effects of such a mean preserving spread of incomes. For instance, does higher loss aversion enhance, or mitigate, the effects of a mean preserving spread of incomes on savings?[18]

Consider mean preserving spreads in the distribution of the time $t = 2$ random income, z. In our experiments, we consider symmetric shocks of the form $\varepsilon_l = -\varepsilon_h$, and each of the shocks occurs with probability 0.5 (i.e., $p = 1 - p = 0.5$), so that $Ez = 0$.

Suppose that we index the size of the shocks by a real-valued parameter $\theta > 0$, so that in the good state, the shock is $\theta\varepsilon_h$ and in the bad state the shock is $\theta\varepsilon_l$:

$$\theta\varepsilon_l = -\theta\varepsilon_h, \tag{4.1}$$

and

$$p = 1 - p = 0.5, \tag{4.2}$$

so that $Ez = 0$. A mean preserving spread requires us to increase the size of θ; it preserves $Ez = 0$ but entails a larger, symmetric, spread of the stochastic time $t = 2$ income. Hence, we define a mean preserving spread of incomes, in our model, as an increase in the parameter θ.

Proposition 3 Consider a mean preserving spread of the time $t = 2$ stochastic income, z, captured by an increase in the size of the parameter θ. The decision maker has Köszegi-Rabin preferences given in (3.1), and suppose that $\mu > 0$.
(a) If $u''' > 0$ (respectively, $u''' < 0$), then optimal savings are increasing (respectively, decreasing) in the mean preserving spread of incomes, i.e., when the parameter θ increases.
(b) The response of optimal savings to a mean preserving spread of the time $t = 2$ income is independent of the parameter of loss aversion.

Discussion of Proposition 3: From Proposition 3(a), optimal savings increases in response to a mean preserving spread of incomes, and this accords well with our intuition. A sufficient condition is $u''' > 0$, which is identical to the sufficient condition for precautionary savings in the classical model (see Proposition 1). However, importantly, from Proposition 3(b), loss aversion does not influence the response of savings to mean preserving spreads of incomes.

Our empirical results show that loss aversion plays a statistically significant role in explaining the savings response to mean preserving spreads of income. Clearly Proposition 3(b) rules out an explanation based on Köszegi-Rabin preferences. One potential explanation is that, unlike in Köszegi-Rabin preferences, gain-loss utility might be non-linear in gains and losses so that the loss aversion term does not wash out in the relevant comparative static results. To see whether this is the case, we consider below, simple prospect theory preferences that allow for non-linear gain-loss utility.[19]

[18] We did not consider the second of these two questions in our original set of theoretical predictions when we took the model to the data. This question arose purely from our empirical findings, and we have gone back to check if this can be explained by our model, or other competing models. The rest of our empirical exercise was strictly informed by the predictions of our theoretical model.

[19] We could also have incorporated the case of non-linear gain-loss utility within Köszegi-Rabin preferences, but the insights are qualitatively the same as the ones we derive below. In any case, Köszegi-Rabin preferences are only stated with linear gain-loss utility.

Consider, the non-linear gain-loss instantaneous utility function, ϕ, in prospect theory, which has empirical as well as axiomatic support (Dhami, 2019, Vol. 1).

$$\phi(c_t, \omega_t) = \begin{cases} (c_t - \omega_t)^\beta & \text{if} \quad c_t \geq \omega_t \\ -\lambda (\omega_t - c_t)^\beta & \text{if} \quad c_t < \omega_t \end{cases}, \; 0 < \beta < 1 \,, t = 1, 2. \tag{4.3}$$

This is identical to the gain-loss function in (2.6) under Köszegi-Rabin preferences, except that it is strictly concave in the domain of gains and strictly convex in the domain of losses. This allows for diminishing sensitivity to gains and losses; the analogue of the concept of diminishing marginal utility.

Suppose that the instantaneous utility function is of the prospect theory form, and the intertemporal utility function is given by

$$\widetilde{U} = \phi(c_1, \omega_1) + E[\phi(c_2, \omega_2)]\,; \; c_1 \in [0, y]\,, c_2 \in X. \tag{4.4}$$

The two terms on the RHS of (4.4) capture the first and second period instantaneous utilities, given in (4.3). Compared to Köszegi-Rabin preferences in (2.4), decision makers under prospect theory derive utility only from gains and losses relative to a reference point, but not from absolute levels of incomes. Using (4.1), (4.2), (4.3), (4.4), and our conditions on the reference points (2.7), (2.8), (2.9), the optimization problem of the decision maker under prospect theory preferences is (recall from (4.2), $p = 1 - p = 0.5$)

$$\widetilde{s} \in argmax \; \widetilde{U} = -\lambda (s)^\beta + \left[(1-p)(s + \theta\varepsilon_h - \omega_2)^\beta - p\lambda (\omega_2 - (s - \theta\varepsilon_h))^\beta \right], \; s \in [0, y]\,. \tag{4.5}$$

We have $c_1 = y - s$, and from (2.7) $\omega_1 = y$, hence, $c_1 - \omega_1 = -s < 0$. We then use the second row of (4.3) to write the first term on the RHS of (4.5). The second term on the RHS of (4.5) takes account of the distribution of the random time $t = 2$ income, z, and the two conditions on the second period reference point, (2.8) and (2.9), before using the appropriate rows in (4.3).

Proposition 4 Consider prospect theory preferences with the non-linear gain-loss utility function in (4.3) and the optimization problem given in (4.5). Suppose that the second order condition holds.
(a) Optimal savings, $\widetilde{s}$, are decreasing in the parameter θ, which captures mean preserving spreads in income.
(b) An increase in the parameter of loss aversion, λ, reduces the response of $\widetilde{s}$ to θ.

Discussion of Proposition 4: From Proposition 4(a), prospect theory preferences predict that a mean preserving spread reduces optimal savings, and from Proposition 4(b), it predicts that loss aversion reduces the savings response to a mean preserving spread.

4.4 Sufficiency of $u'''(x) > 0$ for precautionary savings

In our next result, we show that for loss averse subjects ($\lambda > 1$), a strictly positive third derivative of the utility function, $u'''(x) > 0$, is a sufficient condition for precautionary savings in the presence of gain-loss utility, as in the classical model. However, the condition $u'''(x) > 0$ is neither necessary nor sufficient for precautionary savings for loss tolerant subjects ($\lambda < 1$).

Proposition 5 *(a) Suppose that the decision maker is loss averse ($\lambda > 1$), $\mu > 0$, $Ec_2 > 0$, and $u'' < 0$. Then, the condition $u'''(x) > 0$ is sufficient for the existence of precautionary savings.*
(b) Under the conditions of part (a) if the decision maker is loss tolerant ($\lambda < 1$), then all three outcomes are possible: certainty equivalence, precautionary savings, and reckless undersaving.

However, in the Appendix, we show that in the presence of gain-loss utility ($\mu > 0$), decision makers may exhibit precautionary savings even when we have quadratic utility ($u'''(x) = 0$), provided that in the bad state, consumption relative to the reference point is negative. Thus, the condition $u'''(x) > 0$ is not necessary for precautionary savings, even for loss averse individuals.

5 Experiments and data

5.1 Experimental Design

To test the predictions of our theoretical model, we designed and conducted an incentivized experiment. We developed a two-part, within-subjects, experimental design. Part 1 of the experiment was designed to study savings behavior in a two period model, identical to the one used to derive our theoretical predictions. Part 2 of the experiment was designed to measure subject-specific loss aversion. This allows us to formally test the relationship between loss aversion and the decision to engage in precautionary savings, which is the main focus of the paper. All payoffs were expressed in units of an experimental currency, EC, and converted into real money according to the exchange rate: $1000EC = £1$. All units below are expressed in terms of EC. We now explain both parts in detail.

1. Part 1 (Optimal consumption/savings choice decision): Subjects face a two-period consumption-savings problem where they are asked to allocate a given amount of money now (the time $t = 1$ endowment, y, in the theoretical model) to current consumption, c_1, and future consumption, c_2, in one month's time (time $t = 2$). Thus, the time gap between two successive periods in our experiment is one month. We varied the endowment y in different sub-cases. Whatever amount is allocated to consumption at time $t = 1$, c_1, is paid to the subjects on the same day. The amount saved towards consumption in one month, s, is added to a random income, z, at time $t = 2$. So at time $t = 2$, the subjects receive the income $s + z$.
   At time $t = 1$, the subjects were informed that they will receive a random income at time $t = 2$ that could be either positive or negative (the analogues of ε_h and ε_l in our model), with equal chances; the exact amounts of ε_h and ε_l vary in different sub-cases. If a subject had not saved enough at time $t = 1$ to cover for losses at time $t = 2$ (e.g., $s > 0$ is too low relative to $\varepsilon_l < 0$), she was informed that this amount ($s - \varepsilon_l < 0$) would be subtracted from a guaranteed amount to be paid as a lumpsum at date $t = 2$. The lumpsum of £6 is paid at $t = 2$ in order to ensure that none of our subjects is out of pocket at the end of the experiment. In addition to this lumpsum amount, subjects received a £6 participation, or show-up, fee in the experiment, on the day of the experiment.
   We had three levels of endowments (y = 2000, 3000, 4000) in different sub-cases. We had 4 kinds of shocks, which constituted our random income, z. Three of the shocks were symmetric ($\varepsilon_l = -\varepsilon_h$), where the subject could win or lose an amount with a 50-50 chance (i.e.,

p in our theoretical model equals 0.5); for these symmetric shocks $\varepsilon_h \in [500, 1000, 2000]$, so essentially we have the case of a mean preserving spread in risk as ε_h increases. The fourth shock was an asymmetric shock, where $p = 0.5$ but $\varepsilon_h = 500$ and $\varepsilon_l = -1000$. Thus, subjects had to make a consumption/savings choice at time $t = 1$ for $3 \times 4 = 12$ sub-cases in Part 1. In effect, each sub-case is the use of the strategy method for different levels of incomes, applied to Part 1. The full set of sub-cases is provided in Table 1. One of these sub-cases was played out for real to determine the payoffs from Part 1.

Subjects were encouraged to assume that they have no other outside-the-lab source of income, consumption, or saving and they completed all the tasks without receiving any feedback between rounds.

Table 1: Optimal savings elicitation tasks

Task	Endowment	Positive shock	Negative shock
1	2000	500	-500
2	3000	1000	-1000
3	4000	2000	-2000
4	2000	1000	-1000
5	3000	2000	-2000
6	4000	500	-500
7	2000	2000	-2000
8	3000	500	-500
9	4000	1000	-1000
10	2000	500	-1000
11	3000	500	-1000
12	4000	500	-1000

The Table lists the 12 tasks (or sub-cases) used in Part 1 of the experiment. All payoffs are expressed in terms of experimental currency (EC) used in the experiment, with 1000 EC = £1. In each sub-case there is a 50% probability of the positive and negative shocks.

2. Part 2 (Elicitation of subject-specific loss aversion): The second part of the experiment was designed to elicit a measure of loss aversion at the individual level. We adopted a method similar to Gächter et al. (2022), but used the bisection procedure (Abdellaoui, 2000) to elicit the value of a loss that makes the certainty equivalent of a balanced risk lottery of the form $(x, 0.5; -y, 0.5)$ equal to zero, where $x > 0$ is a gain relative to the reference point, and $-y < 0$ is a loss relative to the reference point.[20] Loss aversion is determined by the ratio $\frac{x}{y}$. In these tasks, the gain, x, is fixed and y is elicited through $k = 1, 2, ..., 6$ lottery choices, where k is the iteration number. In iteration k, subjects had to choose between a lottery of the form $(x, 0.5; -y_k, 0.5)$ and a sure amount of zero, where the amount y_k is determined from the previous $k-1$ choices. However, in the very first lottery choice (i.e., $k = 1$), $-y_1$ is the midpoint of $[-1.4x, -0.1x]$, the feasible interval containing $-y_1$.[21]

[20] We use the standard terminology, so that the lottery $(x, 0.5; -y, 0.5)$ means a 50-50 chance of gaining an amount x and losing an amount y. Our method resembles the switching choice in Gächter et al. (2022), however, our bisection method allows for more variation across subjects.

[21] These amounts are proportional to the gain amount in the lottery, x, such that the upper bound of loss aversion parameter is set at $\frac{x}{0.1x} = 10$. Empirically, the median value of loss aversion is around 2 (Kahneman and Tversky,

If in the first iteration $k = 1$, the lottery is chosen over a sure amount of zero, we make the lottery less attractive in the second iteration, $k = 2$, by having $-y_2$ as the midpoint of the reduced feasible interval $[-1.4x, -y_1]$, otherwise, $-y_2$ is the midpoint of $[-y_1, -0.1x]$; this bisecting process justifies the name of this procedure as the bisection method. The third iteration, $k = 3$, is contingent on the choices in the first two iterations, creating four possibilities. If a subject prefers zero to both lotteries in the first two iterations, $-y_3$ is the midpoint of $[-y_2, -0.1x]$. If a subject prefers lotteries to zero in the first two iterations, then $-y_3 \in [-1.4x, -y_2]$. If a subject first prefers zero at $k = 1$ and then the lottery in the second iteration at $k = 2$, then $-y_3 \in [-y_1, -y_2]$, otherwise, $-y_3$ is the midpoint of $[-y_2, -y_1]$. Hence, the interval containing y shrinks in the remaining choices by replacing the lower or the upper bound of the feasible interval in each iteration based on the subjects' previous choices.

We use three levels of gains $x \in [2000, 3000, 4000]$, the same as the endowments in Part 1 (see Table 1); one of these was randomly chosen to be played out for real. Therefore, subjects faced $3 \times 6 = 18$ iterations in total; one of these iterations was chosen at random to be paid off for real. As noted earlier, any losses were covered by a show-up fee paid on the day.

Finally, we included two non-incentivized questions to gather information on subjects' time preferences. A key element of the modern time discounting literature is the recognition of *preference reversals*, as in models of hyperbolic discounting. By contrast, under exponential discounting, preference reversals cannot arise.[22] We identify present-biased preferences with choices that exhibit preference reversals. We ask subjects to choose between (i) receiving 2500 in one month vs. receiving 2000 today, and (ii) receiving 2500 in 11 months vs. receiving 2000 in 10 months. Subjects exhibit preference reversals if they pick '2000 today' in (i) and they pick '2500 in 11 months' in (ii). We classify such subjects as present-biased.

5.2 Procedures

The subjects were 79 students from a UK Experimental Economics Lab standard subject pool (mostly undergraduate students, 57% female, average age 20.7). The experiment took place in March 2023 and there were in total 6 sessions with 12 to 16 subjects participating in each session. The average payment was £15.65 (£7.71 on the day of the experiment and £7.96 one month later). The sessions lasted 35 minutes, on average, including the time for the instructions and the comprehension test.[23] To cover potential losses, there was a guaranteed show-up fee of £6 paid on the day, and a guaranteed amount of £6 paid in one month.

The experiment was computerized using the LIONESS Lab platform (Giamattei et al. , 2020) and the recruitment took place via ORSEE (Greiner , 2015). Subjects were randomly seated to individual PCs where they could complete the task at their own pace. They were given written instructions, and before they were able to begin the experiment, they had to go through an extensive

2000; Dhami, 2019, Vol. 1).

[22] This is on account of the *stationarity axiom*. However, alternatives to exponential discounting, such as hyperbolic discounting relax this axiom. For a formal treatment, see Dhami (2019, Vol. 3).

[23] Please see the online appendix.

comprehension questionnaire. To ensure that payments could be delivered precisely on time, we completed the payments using Amazon pre-paid cards which were sent to the participants via email immediately after the session and one month later. After completing Parts 1 and 2 of the experiment, subjects were asked to complete a questionnaire to elicit their demographic characteristics. Subjects could not communicate with each other, and their PCs were separated by privacy screens. All subjects participated only once to the experiment.

5.3 Data

We have a sample of 79 students. We elicit loss aversion for three levels of gains $x \in [2000, 3000, 4000]$, the same as the endowments in Part 1 of our experiment.[24] Averaged across all three levels of gains, the mean loss aversion parameter is 1.6571 with a median value of 1.6609; the minimum and maximum values (across all subjects and levels of gains) are, respectively, 0.7196 and 4.1080.

There is no significant difference in the estimated parameter of loss aversion among the three levels of gains. Hence, for each subject, we use the mean value of loss aversion across all three levels of gains, to categorize subjects as either loss averse ($\lambda > 1$) or loss tolerant ($\lambda \leq 1$). This approach is likely to reduce measurement errors and alleviate the assumption of a linear utility function, and is also the approach used in Chapman et al. (2022).

We classify the savings choices of subjects as precautionary savings if the consumption at time $t = 1$ is less than the expected time $t = 2$ consumption, i.e., $c_1 < Ec_2$ (see Definition 1). On average, 46% of the choices exhibit precautionary saving behavior; 36% of the choices indicate reckless undersaving behavior, $c_1 > Ec_2$ (see Definition 1); while the remaining 18% choices exhibit the classic certainty equivalence result $c_1 = Ec_2$. Table 2 shows the number and percentage of observed choices reflecting precautionary saving behavior for each of the 12 sub-cases, which have varying amounts of endowment y and stochastic income, z. The results of the One-Way ANOVA test indicate that there is no significant difference in the mean of savings across the four levels of stochastic income (p-value: 0.971).

Table 2: Precautionary Saving behavior

	Stochastic Income			
Endowment	(500,0.5; -500,0.5)	(1000,0.5; -1000,0.5)	(2000,0.5; -2000,0.5)	(500,0.5; -1000,0.5)
2000	26 (33%)	32(41%)	45 (57%)	35 (44%)
3000	32(41%)	34(43%)	54(68%)	37(47%)
4000	37(47%)	32(41%)	40(51%)	33(42%)

Table shows the number and percentage of subjects engaging in precautionary saving.

Recall that the variable 'Shock' refers to the value taken by the random income at time $t = 2$, and it has 4 categories (see Table 2). Ignore for the moment, the last category, which is an asymmetric shock. For the remaining three symmetric shocks, where the expected value of stochastic income is zero in all cases, and the probability of the bad states is fixed at 0.5, we have the following three cases. The variable 'Shock' takes the value 0 for the reference category with an income of 500 in the good state and -500 in the bad state; the value 1 for a symmetric shock of 1000; and the value 2 for a symmetric shock of 2000.

[24] Recall from the description of Part 2 of our task, above, that x refers to the gain in the lottery $(x, 0.5; -y, 0.5)$.

As the absolute value of the shock increases, we have a mean preserving spread of the time $t = 2$ random income distribution. Most reasonable decision theories will require savings to increase with an increase in the absolute value of the shock, and this also serves as a consistency check on the data. In these three cases we have, respectively, the following mean values of savings measured in EC: 1552.312 (shock value 0), 1639.19 (shock value 1), and 1854.599 (shock value 2). Thus, subjects in our experiments respond to higher income uncertainty by saving more, as one might expect. The traditional explanation for this phenomenon is risk aversion. However, we show later that this is on account of loss aversion.

6 Regression Results

From Proposition 2(i), loss aversion 'directly' reduces savings. As far as we know, we are the first to test this prediction in an explicitly dynamic model, where individual-specific loss aversion is directly measured.[25] As noted in Corollary 1, a direct implication is that the precautionary savings of loss averse subjects ($\lambda > 1$) are likely to be lower than the precautionary savings of loss tolerant subjects ($\lambda < 1$). We test these predictions in this section.

Another important set of predictions that we test in this section relate to the effects of mean preserving spreads of time $t = 2$ income on optimal savings. We show that Köszegi-Rabin preferences (i) explain well the effects of mean preserving spreads on savings (Proposition 3(a)), but (ii) are unable to explain why loss aversion strengthens the response of savings to mean preserving spreads of income (Proposition 3(a)). The predictions of prospect theory on the optimal response of savings to mean preserving spreads in income (Proposition 4(a)) or on the mediating role of loss aversion in this response (Proposition 4(b)) are not supported by the data.

6.1 Determinants of Precautionary Savings

Recall that an individual engages in precautionary saving if $c_1^* < Ec_2^*$ and reckless undersavings if $c_1^* > Ec_2^*$. Therefore, our dependent variable is binary and we employ the following logit model to analyze the determinants of precautionary saving behavior:

$$P(Y = 1|X) = P(\beta X + u > 0) = F(\beta X) = \frac{1}{1 + \frac{1}{e^{\beta X}}}, \tag{6.1}$$

where $Y = 1$ indicates precautionary saving behavior ($c_1^* < Ec_2^*$) and $Y = 0$, its absence. X is a vector of explanatory variables and β is a vector of coefficients. The explanatory variables used in (6.1), with the corresponding names given in Table 3, and the basic data on the individual categories, are as follows.

- 'Loss aversion': Dummy variable that takes the value 1 if the subject is loss averse ($\lambda > 1$) and 0 if the subject is loss tolerant ($\lambda \leq 1$). 63/79 (80%) of subjects are loss averse.[26]
- 'Present bias': Dummy variable that takes the value 1 if the subject is present biased and 0 otherwise (see Section 5 for an explanation). 22/79 (28%) of subjects are present biased.

[25] The only exception that we are aware of is Ibanez and Schneider (2023) who use observational data. However, in the introduction, we have noted the extensive differences of our paper from theirs.

[26] We follow here the distinction between loss averse and loss tolerant subjects in Chapman et al. (2022). With a continuous measure of loss aversion, the results are qualitatively similar, although it reduces the statistical significance of loss aversion.

• 'Age' gives the self-reported age of subjects. The minimum and maximum age is, respectively, 18 and 35; the mean and median age is, respectively, 20.72 and 19; and the standard deviation is 3.36.

• 'Gender' is a dummy variable for gender and takes the value 1 for male and 0 for female. 34/79 subjects (43%) are males and 45/79 subjects (57%) are females.

• 'Education' is a dummy variable. It equals 1 for Masters/PhD students, and 0 otherwise. 62/79 subjects (78%) are undergraduates, 16/79 (20%) are master students and there is 1 PhD student.

• 'Income' is a self declared outside-the-lab monthly expense that is used as a proxy for the real world income of the subjects. The mean income is £425, and the median is £300. The standard deviation is £334.

• 'Time' indicates the length of time taken for the completion of the experiment.

• 'Endowment' is a categorical variable that captures the subject's endowment in each question. It takes the value 0 for the reference category when the endowment is 2000, 1 for an endowment of 3000, and 2 for an endowment of 4000 (this corresponds to distinct values of y, the first period income endowment in our model).

• 'Shock' is a categorical variable that indicates the stochastic income at time $t = 2$ (see details in Table 2 and it corresponds to stochastic income z in our model). It takes the value 0 for the reference category when the expected value of stochastic income is zero, with an income of 500 in the *good state* and -500 in the *bad state*. The probability of the *good* and *bad* states is fixed at 0.5 in all cases. It takes the value 1 for a symmetric shock of 1000, 2 for a symmetric shock of 2000, and 3 for an asymmetric shock with a negative expected value, where the income in the *good state* is 500 and in the *bad state* is -1000.

In Table 3, we present the logit regression results; the dependent variable is $P(Y = 1|X)$ given in (6.1). In the second column, we present the coefficient estimates; the third column gives the Odds-Ratio; and the last column gives the marginal effects from the logit model.

From Table 3, loss aversion decreases the odds of precautionary saving by 52% $((1-0.448)\times 100)$. In other words, the odds of precautionary saving behavior are 0.448 times lower for loss averse individuals compared to loss tolerant ones (keeping all other predictors constant). This finding aligns with our theoretical prediction that loss tolerant subjects are more likely to engage in precautionary savings (Corollary 1). The last column of Table 3 reports marginal effects. Being loss averse, relative to loss tolerant, decreases the probability of precautionary saving behavior by almost 20%.

Present-biased individuals exhibit a reduced likelihood of engaging in precautionary saving behavior.[27] The effect of present bias is comparable to that of loss aversion, and both effects are significant at the 1% level. This is an important result and speaks to the growing literature between risk and time preferences; for a survey, see Dhami (2019, Vol. 3).

[27] Recall our definition of present bias, which is consistent with a violation of the stationarity axiom for the axioms of rationality under time discounting (Dhami, 2019, Vol. 3). Hence, one cannot simply derive the theoretical implications of present bias by introducing a simple discount factor, δ , to discount future utilities. At a minimum, this would require setting up a model of quasi-hyperbolic discounting, but that requires a minimum of three time-dated values of consumption.

Table 3: Logit Regression results

	Dependent variable is a dummy for pre cautionary saving		
	Estimates	Odds-Ratio	Marginal Effects
Loss aversion	-0.803^{***}	0.448^{***}	-0.198^{***}
	(0.194)	(0.087)	(0.046)
Present bias	-0.801^{***}	0.449^{***}	-0.192^{***}
	(0.165)	(0.074)	(0.037)
Age	0.060^{*}	1.062^{*}	0.014^{*}
	(0.033)	(0.035)	(0.008)
Gender	-0.493^{***}	0.611^{***}	-0.121^{**}
	(0.155)	(0.095)	(0.038)
Education	0.334	1.397	0.083
	(0.233)	(0.325)	(0.058)
Income	-0.001^{***}	0.999^{***}	-0.0002^{***}
	(0.0002)	(0.0002)	(0.0000)
Time	0.002^{***}	1.002^{***}	0.0003^{**}
	(0.001)	(0.001)	(0.0001)
Endowment=3000	0.273	1.314	0.068
	(0.170)	(0.223)	(0.042)
Endowment=4000	0.058	1.060	0.014
	(0.170)	(0.180)	(0.042)
Shock=1	0.058	1.060	0.014
	(0.197)	(0.208)	(0.049)
Shock=2	0.832^{***}	2.298^{***}	0.204^{***}
	(0.197)	(0.453)	(0.047)
Shock=3	0.192	1.211	0.048
	(0.196)	(0.237)	(0.049)
Constant	-1.263^{*}	0.283^{*}	
	(0.663)	(0.188)	
Observations	948	948	948
Log Likelihood	−596.391		
Akaike Inf. Crit.	1,218.781		

Note: *p<0.1; **p<0.05; ***p<0.01. Robust standard errors are in parentheses.

Compared to females, males are less likely to engage in precautionary savings; being male decreases the odds of such behavior by 39%. This result speaks to the large literature on relatively greater risk-seeking and overconfidence among men, relative to women; for a literature survey, see Dhami (2016). Older individuals and those who spend more time deliberating on saving decisions are more likely to engage in precautionary saving, which is a result that aligns with expectations.[28]

Recall from Table 1 that in 3 out of 4 cases, we had symmetric shocks to random income at time $t = 2$, so that the expected value of the shock equals 0. However, the magnitude of the shocks varies; and the negative shock takes respective values, −500, −1000, −2000 in the first three sub-cases. Interestingly, it appears that the magnitude of the negative shock in the bad state has more influence than the expected value of the stochastic income. When the size of the loss increases to 2000 (Shock equals 2) compared to the reference category of Shock (a loss of 500 in the bad state), the odds of precautionary saving increase by 130%. This also accords well with one's intuition.

Similar results are observed within subsets of our data. Table 4 presents the logit regression results within each time $t = 1$ endowment level, as well as the pooled results across all endowment levels. In all of these regressions, the coefficient of loss aversion is consistently negative and significant at the 5% level.

Table 4: Determinants of precautionary saving behavior within each endowment

	Dependent variable is a dummy for pre cautionary saving			
	(Endowment of 2000)	(Endowment of 3000)	(Endowment of 4000)	(Pooled)
Loss aversion	-0.760^{**}	-0.687^{**}	-0.929^{**}	-0.781^{***}
	(0.357)	(0.336)	(0.377)	(0.205)
Present bias	-0.873^{***}	-0.770^{***}	-0.709^{**}	-0.778^{***}
	(0.283)	(0.279)	(0.289)	(0.163)
Age	0.045	0.063	0.069	0.058^{*}
	(0.058)	(0.060)	(0.066)	(0.035)
Gender	−0.166	-0.454^{*}	-0.838^{***}	-0.479^{***}
	(0.264)	(0.260)	(0.275)	(0.152)
Education	0.309	0.300	0.387	0.325
	(0.370)	(0.378)	(0.371)	(0.212)
Income	-0.001^{***}	−0.001	-0.001^{**}	-0.001^{***}
	(0.0004)	(0.0004)	(0.0004)	(0.0002)
Time	0.001	0.002	0.002^{*}	0.002^{***}
	(0.001)	(0.001)	(0.001)	(0.001)
Constant	−0.899	−0.979	−0.704	−0.860
	(1.112)	(1.041)	(1.122)	(0.627)
N	316	316	316	948

Note: *p<0.1; **p<0.05; ***p<0.01. Robust standard errors are in parentheses.

We find support for another prediction of our model, which suggests that among subjects in the precautionary saving category, those who are less loss averse show a higher difference between expected consumption in period 2, Ec_2^*, and current consumption, c_1^*; this follows from the results derived in the lead up to Corollary 1. This indicates a negative correlation between loss aversion and the difference between expected consumption in period 2 and current consumption ($Ec_2^* - c_1^*$).

[28] However, we do not find any significant association between present bias/loss aversion and the two variables, age and deliberation time. Hence, there are likely to be other cognitive and/or neural channels that are associated with more caution in the presence of these variables.

For choices belonging to the precautionary saving category, the Pearson's correlation between loss aversion and $Ec_2^* - c_1^*$ is -0.087 ($p - value = 0.0675$). On the other hand, for choices belonging to reckless undersaving or certainty equivalence, this correlation is 0.075 ($p - value = 0.0869$).

6.2 Determinants of savings

In Table 5, the dependent variable is the actual savings by individuals. We present two different models. Model 1 contains the same explanatory variables as those used in Table 3. Model 2 adds interaction terms between loss aversion and the variable 'Shock' in order to explain why a mean preserving spread of incomes elicits greater savings.[29]

The general results parallel those on precautionary savings. Consider the estimates in Model 2. Loss averse subjects save, on average, 423 units less than loss tolerant subjects. This confirms the prediction in Proposition 2(i) that an increase in loss aversion 'directly' reduces savings. The effect of loss aversion on savings is larger than the impact of present bias on savings, which decreases savings by 267 units (while holding all other predictors constant). Both effects are statistically significant at the 1% level. In comparison to females, males tend to save less; and being male is associated with a decrease in savings by 174 units. Older individuals and those who spend more time deliberating on the saving decisions tend to save slightly more. Higher levels of initial time $t = 1$ endowments have a substantial positive and effect on saving that is significant at the 1% level. Thus, mean preserving spreads of income elicit greater savings.

When comparing different categories of shocks, we again get results similar to those for precautionary savings. Compared to the reference category of shock (Shock = 0), an increase in the size of the loss to 2000 units (Shock = 2) leads to a savings increase of 302 units in Model 1, and this effect is significant at the 1% level. This is consistent with the predictions of the optimal response of savings to mean preserving spreads of income (Proposition 3(a)). Furthermore, in Model 1, for an asymmetric shock with a negative expected value (Shock = 3), savings increase by 117 units relative to the reference category, but this result is only marginally significant.[30]

Once we add the interaction terms in Model 2, these results change. None of the shock variables, by themselves, have statistical significance anymore. However, the following two findings are of interest.

1. The interaction term "Loss aversion: Shock=2" is now positive and significant at the 5% level. In other words, being loss averse relative to being loss tolerant, increases savings by $481 - 423 = 58$ units when Shock=2, relative to the reference category where Shock=0. Thus, being loss averse, relative to loss tolerant, sharpens the response of savings to mean preserving spreads of time $t = 2$ random income.

2. Conditional on a subject being loss averse, Shock=2 induces a decision maker to save an extra 481 units relative to the reference category of Shock=0 and this is statistically significant.

[29] When we interact loss aversion with the variable 'Shock' in Table 3, none of the coefficients is significant, hence, we do not report those results. But these results are available on request.

[30] Note that our predictions on the optimal response of savings to mean preserving spreads do not cover the case of Shock = 3 because in this case the mean and the variance of the shocks changes simultaneously.

Table 5: OLS regressions on savings

	Dependent variable is saving:	
	(1)	(2)
Loss aversion	-270.347^{***}	-422.669^{***}
	(76.012)	(142.910)
Present Bias	-266.797^{***}	-266.797^{***}
	(56.119)	(56.108)
Age	33.112^{**}	33.112^{**}
	(14.579)	(14.289)
Gender	-174.455^{***}	-174.455^{***}
	(54.323)	(53.843)
Education	92.905	92.905
	(71.514)	(70.945)
Income	-0.199^{**}	-0.199^{**}
	(0.083)	(0.082)
Time	0.323	0.323^{*}
	(0.197)	(0.195)
Endowment=3000	526.171^{***}	526.171^{***}
	(50.542)	(50.265)
Endowment=4000	$1,021.136^{***}$	$1,021.136^{***}$
	(61.815)	(61.494)
Shock=1	86.878	44.583
	(69.884)	(185.289)
Shock=2	302.287^{***}	-81.208
	(72.208)	(200.379)
Shock=3	117.435^{*}	57.333
	(70.268)	(181.639)
Loss aversion: Shock=1		53.036
		(199.403)
Loss aversion: Shock=2		480.891^{**}
		(213.588)
Loss aversion: Shock=3		75.365
		(196.462)
Constant	663.900^{**}	785.373^{***}
	(260.856)	(283.473)

Note: *p<0.1; **p<0.05; ***p<0.01. Robust standard errors are in parentheses.

Both effects have reasonably high magnitudes. Thus, a mean preserving spread in income may be said to induce a *loss averse-hedging motive*. The traditional explanation for this phenomenon is in terms of risk aversion. Our results indicate that ignoring loss aversion may lead to misleading results and an over emphasis on the importance of risk aversion.

The loss averse-hedging motive is difficult for economic theory to explain. Köszegi-Rabin preferences cannot explain this finding (see Proposition 3); and by default, standard expected utility theory cannot either. Standard prospect theory preferences in fact make the opposite prediction to the one that we find in our data (see Proposition 4). Loss aversion is not a part of the repertoire of several other behavioral models that might be used, such as models of salience, limited attention, and hyperbolic discounting. Hence, these models cannot provide a resolution to this empirical finding either. It is possible that models of bounded rationality and mental accounting might provide a potential resolution. For instance, decision makers might have a mental account for consumption from stochastic incomes and use simple rules of thumb and loss aversion to minimize the variance of consumption. But using such models, and exploring their underlying transmission mechanisms to explain our data, lies outside the scope of this paper.

7 Conclusions

Microeconomic theory implicates risk aversion and loss aversion as the key determinants of hedging against future income risk in dynamic models. Macroeconomic theory implicates precautionary savings as the key determinant. We construct a bare bones macroeconomic model with future income uncertainty and show that loss aversion and precautionary motives are related.

Our theoretical model predicts that loss averse decision makers save less, and are less likely to engage in precautionary savings. Our empirical results are consistent with this prediction. We also show that optimal savings increase in response to mean preserving spreads of income, and this is also consistent with our data. However, loss averse decision makers are found to respond even more strongly to a mean preserving spread of the random future income. We term this as the *loss averse-hedging motive* relative to the standard risk-hedging motive in classical theory. However, it is difficult for economic theory to explain this finding, and we show that it cannot be 'directly' explained by Köszegi-Rabin preferences, prospect theory preferences, salience theory, models of limited attention, or by hyperbolic discounting, among others. Replicating this finding, and if established, explaining this finding by using the relevant theory can be a fruitful avenue for future research.

We take account of the effects of loss aversion on current and future consumption in making our predictions on the effects of loss aversion on savings. In so doing, our methods are consistent with the work of Thaler and Benartzi (2004). However, a body of theoretical work takes account of the effects of loss aversion only on future consumption and predicts that loss aversion will increase current savings. This prediction is not consistent with our data. We also show that the effects of risk aversion on savings are ambiguous in our model. But the effects of present bias, albeit a bit smaller in magnitude than the effects of loss aversion, are also statistically significant in reducing savings.

Appendix

Proofs of Results

Proof of Proposition 1: Substitute $\mu = 0$ in (3.6). At an interior solution, and using the two budget constraints, (2.1) and (2.2), we get

$$u'(c_1) = E\left[u'(c_2)\right]. \tag{7.1}$$

Suppose that the utility function satisfies the restriction $u'''(x) > 0$, for all $x \in X$. Then, u' is a strictly convex function for all $x \in X$. From Jensen's inequality, it follows that $u'(Ec_2) < E\left[u'(c_2)\right]$. Thus, using (7.1), we get

$$u'(Ec_2) < u'(c_1). \tag{7.2}$$

By assumption, $u''(x) < 0$ for all $x \in X$, thus, it follows from (7.2) that $c_1 < Ec_2$ (precautionary savings). If, on the other hand, $u'''(x) < 0$ for all $x \in X$, then Jensen's inequality implies that $u'(Ec_2) > E\left[u'(c_2)\right]$, so the condition $u''(x) < 0$ for all $x \in X$ implies that $c_1 > Ec_2$ (reckless undersaving). ■

Proof of Proposition 2: (i) Applying the implicit function theorem to (3.6), we get

$$\frac{\partial s}{\partial \lambda} = -\mu\left(-\frac{\partial^2 U}{\partial s^2}\right)^{-1}(1-p) < 0.$$

(ii) Applying the implicit function theorem to the first order condition in (3.6), we get

$$\frac{\partial s}{\partial \mu} = \left(-\frac{\partial^2 U}{\partial s^2}\right)^{-1}\left[(1-p)(1-\lambda)\right].$$

It follows that if the decision maker is loss averse, $\lambda > 1$, then $\frac{\partial s}{\partial \mu} < 0$; and if the decision maker is loss tolerant, $\lambda < 1$, then $\frac{\partial s}{\partial \mu} > 0$.

(iii) For the CRRA utility function, $u(c) = \frac{1}{1-\gamma}c^{1-\gamma}; \gamma > 0$, $\gamma \neq 1$, the foc in (3.6) is

$$-\left(y-s\right)^{-\gamma} - \mu\lambda + \left[p(s+\varepsilon_l)^{-\gamma} + (1-p)\left(s+\varepsilon_h\right)^{-\gamma}\right] + \mu\left[p\lambda + (1-p)\right] = 0.$$

Implicitly differentiating, we get

$$\frac{\partial s}{\partial \gamma} = \left(-\frac{\partial^2 U}{\partial s^2}\right)^{-1}\left[\left(y-s\right)^{-\gamma} ln\left(y-s\right) - p(s+\varepsilon_l)^{-\gamma} ln(s+\varepsilon_l) - (1-p)\left(s+\varepsilon_h\right)^{-\gamma} ln(s+\varepsilon_h)\right] \gtreqless 0.$$

■

Proof of Corollary 1: Suppose that subjects in our experiments discount the time $t = 2$ utility by a discount factor $0 < \delta \leq 1$. The case $\delta = 1$ subsumes our model and all results below also hold for the case $\delta = 1$. Consider the variable $\chi\left(\lambda\right) = c_1^* - \delta Ec_2^*$. Substituting $c_1^* = y - s^*$ and $Ec_2^* = s^* + \overline{z}$, we get $\chi = \left(y - s^*\right) - \delta\left(s^* + \overline{z}\right)$, or

$$\chi\left(\lambda\right) = y - (1+\delta)s^*\left(\lambda\right) - \delta\overline{z},$$

where we have suppressed dependence of savings on factors other than loss aversion, λ. Since $s^*\left(\lambda\right)$ is continuously differentiable in λ (theorem of the maximum), χ is a continuously differentiable function of λ. Let $\lambda = \widehat{\lambda}$ be defined such that $\chi\left(\widehat{\lambda}\right) = 0$, i.e, when $\lambda = \widehat{\lambda}$, we have $c_1^* = \delta E c_2^*$ (certainty equivalence). We have

$$\frac{\partial \chi}{\partial \lambda} = -(1+\delta)\frac{\partial s^*}{\partial \lambda} > 0, \tag{7.3}$$

where the sign of (7.3) follow directly from Proposition 2(i). This also holds if, as in our theoretical model, $\delta = 1$. Hence, using the continuity and monotonicity of χ, we have that for all $\lambda < \widehat{\lambda}$, it must be the case that $\chi(\lambda) < 0$, or $c_1^* < \delta E c_2^*$ (precautionary savings), and for all $\lambda > \widehat{\lambda}$, it must be the case that $\chi(\lambda) > 0$, or $c_1^* > \delta E c_2^*$ (reckless undersaving). ■

Proof of Proposition 3: (a) Using (4.1), (4.2), we can write the first order condition in (3.6) for an interior solution as

$$\frac{\partial U}{\partial s} = \left[-u'(y-s^*) - \mu\lambda + \mu\left(p\lambda + (1-p)\right)\right] + p\left[u'(s^* - \theta\varepsilon_h) + u'(s^* + \theta\varepsilon_h)\right] = 0.$$

Given our assumptions, the optimal savings funtion is continuously differentiable, hence, using the implicit function theorem, we get

$$\frac{\partial s^*}{\partial \theta} = \left(-\frac{\partial^2 U}{\partial s^2}\right)^{-1} p\varepsilon_h\left[-u''(s^* - \theta\varepsilon_h) + u''(s^* + \theta\varepsilon_h).\right] \tag{7.4}$$

The sign is determined by the term in the square brackets on the RHS. Since $u'' < 0$, the first term in the square brackets is positive and the second is negative, thus, the signs of $\frac{\partial s^*}{\partial \theta}$ is, in general, indeterminate. However, if $u''' > 0$, then $u''(s^* - \theta\varepsilon_h) < u''(s^* + \theta\varepsilon_h)$ or

$$0 < u''(s^* + \theta\varepsilon_h) - u''(s^* - \theta\varepsilon_h),$$

which implies that the term in the square brackets is positive, so $\frac{\partial s^*}{\partial \theta} > 0$. Otherwise, if $u''' < 0$, then the term in the square brackets is negative, and we have $\frac{\partial s^*}{\partial \theta} < 0$.

(b) The RHS of (7.4) is independent of the parameter of loss aversion, λ. Hence, $\frac{\partial}{\partial \lambda}\left(\frac{\partial s^*}{\partial \theta}\right) = 0$. ■

Proof of Proposition 4: Using (4.5), the relevant first order condition (the analogue of the first order condition for Köszegi-Rabin preferences in (3.6)) is

$$\frac{\partial \widetilde{U}}{\partial s} = -\lambda\beta(s)^{\beta-1} + (1-p)\beta(s + \theta\varepsilon_h - \omega_2)^{\beta-1} + p\lambda\beta\left(\omega_2 - (s - \theta\varepsilon_h)\right)^{\beta-1} = 0. \tag{7.5}$$

Suppose that the second order condition holds, so that at the optimal solution, $\frac{\partial^2 \widetilde{U}}{\partial s^2} < 0$.[31] Under the given assumptions, $\widetilde{s}$ is continuously differentiable. Hence, using the implicit function theorem, we get

$$\frac{\partial \widetilde{s}}{\partial \theta} = \left(-\frac{\partial^2 \widetilde{U}}{\partial s^2}\right)^{-1}\left[(1-p)\varepsilon_h\beta(\beta-1)(s + \theta\varepsilon_h - \omega_2)^{\beta-2} + p\lambda\varepsilon_h\beta(\beta-1)\left(\omega_2 - (s - \theta\varepsilon_h)\right)^{\beta-2}\right] < 0. \tag{7.6}$$

[31] We have

$$\frac{\partial^2 \widetilde{U}}{\partial s^2} = \beta(\beta-1)\left[-\lambda(s)^{\beta-1} + (1-p)(s + \theta\varepsilon_h - \omega_2)^{\beta-2} - p\lambda\left(\omega_2 - (s - \theta\varepsilon_h)\right)^{\beta-2}\right].$$

The satisfaction of the second order condition requires the second term inside the square brackets to be large enough relative to the other two terms.

(b) Differentiating (7.6) with respect to the parameter of loss aversion, λ, we get

$$\frac{\partial}{\partial \lambda}\left(\frac{\partial \widetilde{s}}{\partial \theta}\right) = \left(-\frac{\partial^2 \widetilde{U}}{\partial s^2}\right)^{-1} p\varepsilon_h \beta (\beta - 1)(\omega_2 - (s - \theta\varepsilon_h))^{\beta-2} < 0. \blacksquare \tag{7.7}$$

Proof of Proposition 5: (a) Using the budget constraints, (2.1), (2.2), we can rewrite (3.6),

$$E\left[u'(c_2)\right] = u'(c_1) + \mu\lambda - \mu\left(p\lambda + (1-p)\right). \tag{7.8}$$

If $u'''(x) > 0$, then u' is a strictly convex function, thus, it follows from Jensen's inequality that

$$u'(Ec_2) < E\left[u'(c_2)\right]. \tag{7.9}$$

We have $\mu\lambda - \mu\left(p\lambda + (1-p)\right) = \mu(1-p)(\lambda - 1)$. For loss tolerant subjects $\lambda < 1$. In this case we have from (7.8), (7.9) that

$$u'(Ec_2) < u'(c_1) + \mu(1-p)(\lambda - 1) < u'(c_1). \tag{7.10}$$

Since $Ec_2 > 0$ we have, by assumption, that $u'' < 0$. Hence, from (7.10), we get $c_1 < Ec_2$, which implies that a loss averse decision maker engages in precautionary savings (see Definition 1).

(b) If the decision maker is loss tolerant, $\lambda < 1$, then from (7.10) $c_1 \gtreqless Ec_2$, all three outcomes are possible: certainty equivalence, precautionary savings, and reckless undersaving. $\blacksquare$

An analysis of precautionary savings when consumption in the bad state is negative

Negative levels of consumption

The assumed concavity of u in most macroeconomic models requires the assumption of non-negative consumption levels. However, there are important reasons why we may also need to consider the possibility of negative consumption (or negative consumption relative to a reference point).

1. The shock in the bad state, ε_l, might be so severe that it drives the absolute level of consumption in the bad state to a negative value. Indeed, households might have to sell assets to maintain positive consumption in such states of the world, or rely on social or public support.

2. Due to mental accounting, individuals might not aggregate all sources of income and wealth when they write down their budget constraints in (2.1) and (2.2) (Thaler, 1985, 1999). An important feature of mental accounting is that money itself might not be fungible across different mental accounts; for surveys and implications for the lifecycle model, see Thaler (2015) and Dhami (2020, Vol. 5). Different items of expenditure or different sources of incomes are classified by individuals to be in different mental accounts, e.g., an entertainment/holiday/vacation account, a children's education account, and a household durables account. In the mental accounting view, individuals might engage in intertemporal consumption smoothing separately for different mental accounts. It is, thus, possible that some mental accounts might go into the red (negative consumption) and this is aversive to individuals (Prelec and Loewenstein, 1998).

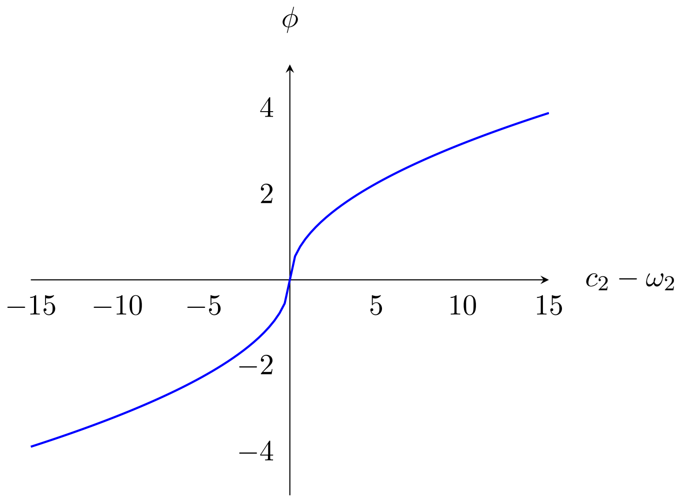

Figure 1: The utility function when second period consumption can be positive and negative.

If we allow for the possibility that second period consumption is negative, $c_2 < 0$, then, we can no longer assume $u'' < 0$ over the entire domain of consumption values because it violates diminishing marginal utility of consumption. The appropriate assumption to make in order to ensure diminishing marginal utility of consumption in the domain $c_2 < 0$, is $u'' > 0$. Such an analysis already lies at the heart of prospect theory because it allows for 'consumption relative to the reference point' to be either positive or negative; indeed it is over this domain that the utility function is defined. For this reason, we provide the next example in terms of a prospect theory utility function, although it can be given in terms of a standard utility function u over absolute levels of consumption, with $c_2 < 0$, invoking the arguments that we have given above; and we use a standard utility function in Section 7.

Example 2 *Consider the following prospect theory utility function, $\phi : \Re \to \Re$ for second period consumption that has empirical support and has axiomatic foundations (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992; al-Nowaihi et al., 2008).*

$$\phi\left(c_2 - \omega_2\right) = \begin{cases} \left(c_2 - \omega_2\right)^{\alpha} & if \quad c_2 - \omega_2 \geq 0 \\ -\lambda\left(-\left(c_2 - \omega_2\right)\right)^{\alpha} & if \quad c_2 - \omega_2 < 0 \end{cases}, \tag{7.11}$$

where $\lambda > 1$ is the parameter of loss aversion and $0 < \alpha < 1$; while c_2 is second period consumption and the reference point is ω_2. In effect, the function ϕ is a non-linear version of the gain-loss utility function g, defined in (2.6). In the domain $c_2 - \omega_2 < 0$, we have $\phi' = \alpha\lambda\left(-\left(c_2 - \omega_2\right)\right)^{\alpha-1} > 0$ and $\phi'' = -\alpha(1-\alpha)\lambda\left(-\left(c_2 - \omega_2\right)\right)^{\alpha-2} > 0$. Thus, the utility function is convex in the negative domain. Figure 1 illustrates the case $\alpha = 0.5$.

The non-necessity of $u'''(x) > 0$ for precautionary savings

We now show that when consumption takes potentially negative values, the condition $u'''(x) > 0$ in Proposition 5 is a sufficient condition, but not a necessary condition for precautionary savings for loss averse individuals in the presence of gain loss utility ($\mu > 0$). Consider the case of quadratic utility, where $u'''(x) = 0$. From Section 3.1, under these conditions in the classical model ($\mu = 0$), certainty equivalence must hold. So there is no precautionary savings in the classical model. We now ask the question: Under precisely these conditions, can there be a precautionary savings motive for Köszegi-Rabin preferences and for loss averse individuals?

Using (3.7) define

$$f(c) = c - \frac{a}{2}c^2,\ a > 0,\ c \geq 0. \tag{7.12}$$

We now wish to extend quadratic utility to the case when consumption takes both positive and negative values, as follows

$$u(c) = \begin{cases} f(c) & if \quad c \geq 0 \\ -f(-c) & if \quad c < 0 \end{cases}, \tag{7.13}$$

where $f(-c) = (-c) - \frac{a}{2}(-c)^2$. The first row of (7.13) gives a standard quadratic function when $c \geq 0$ and the second row of (7.13) gives a quadratic function when $c < 0$. Our definition of utility in the negative domain follows the same convention as in prospect theory, outlined in Example 2. It relies on the idea of diminishing marginal utility, or diminishing sensitivity, as we show next.

Using (7.13), the marginal utilities in both domains are given by[32]

$$u'(c) = \begin{cases} 1 - ac & if \quad c \geq 0 \\ 1 + ac & if \quad c < 0 \end{cases}. \tag{7.14}$$

In order to demonstrate diminishing marginal utility, as we move out from the origin in either direction, differentiate the marginal utilities in (7.14) again to get

$$u''(c) = \begin{cases} -a < 0 & if \quad c \geq 0 \\ a > 0 & if \quad c < 0 \end{cases}. \tag{7.15}$$

Using (7.15), $u'''(c) = 0$ in both domains. Noting that $Eu'(c_2) = p[1 + a(s + \varepsilon_l)] + (1 - p)[1 - a(s + \varepsilon_h)]$, and using the budget constraints $c_1 + s = y$ and $c_2 = s + z$, the Euler equation in (3.6), when $\mu > 0$, can be rewritten as

$$\frac{\partial U}{\partial s} = -u'(y - s) - \mu\lambda + E\left[u'(s + z)\right] + \mu\left(p\lambda + (1 - p)\right) = 0.$$

Or,

$$1 - a(y - s) + \mu\lambda = p[1 + a(s + \varepsilon_l)] + (1 - p)[1 - a(s + \varepsilon_h)] + \mu[p\lambda + (1 - p)] \tag{7.16}$$

Let us consider the following simple case of symmetric balanced risk.

$$p = \frac{1}{2}; \varepsilon_h = \varepsilon > 0; \varepsilon_l = -\varepsilon, \tag{7.17}$$

and the following simplifying parameter values

$$\mu = a = 1. \tag{7.18}$$

Substitute (7.17), (7.18) in (7.16), we get the optimal savings

$$s^* = y - \varepsilon + \frac{1}{2}(1 - \lambda). \tag{7.19}$$

[32]When $c < 0$, $u(c) = -f(-c) = c + \frac{a}{2}(-c)^2$. We have $u'(c) = 1 + \frac{a}{2}2(-c)(-1)$ or $u'(c) = 1 + ac$. Differentiating again, we get $u''(c) = a > 0$, which is convex. The same calculations apply to the prospect theory utility function in losses which is also convex (see Example 2), although the domain in that case is consumption relative to a reference point.

From (7.19), loss aversion reduces optimal savings (see also Proposition 2(i)). Substituting (7.19) in (2.1), we have

$$c_1^* = y - s^* = \varepsilon - \frac{1}{2}(1 - \lambda). \tag{7.20}$$

We have $Ec_2^* = s^* + \overline{z}$, and $\overline{z} = 0$ (symmetric balanced risk). Using (7.19), we have

$$Ec_2^* = s^* = y - \varepsilon + \frac{1}{2}(1 - \lambda). \tag{7.21}$$

Comparing (7.20) and (7.21), we get

$$c_1^* < Ec_2^* \Leftrightarrow \lambda < 1 + (y - 2\varepsilon), \tag{7.22}$$

where

$$y - 2\varepsilon \lesseqgtr 0.$$

From Definition 1 precautionary savings arises when $c_1 < Ec_2$ and reckless undersavings arises when $c_1 > Ec_2$. The condition in (7.22) shows how these two cases can arise in the presence of gain-loss utility if loss aversion is low enough but it can be greater than 1 (i.e., loss averse individuals). There are two differences in our example in this section from the traditional analysis in Section 3.1. (1) We have a reference point for consumption, and we allow for the possibility that in the bad state, consumption relative to the reference point might be negative. (2) There is gain-loss utility and loss aversion.

References

[1] Abdellaoui, M. (2000). Parameter-Free Elicitation of Utility and Probability Weighting Functions. Management Science 46(11): 1497-1512.

[2] Aizenman, J. (1998). Buffer stocks and precautionary savings with loss aversion. Journal of International Money and Finance 17 (6), 931–947.

[3] Bayer, C., R. Lütticke, L. Pham-Dao, and V. Tjaden (2019). Precautionary savings, illiquid assets, and the aggregate consequences of shocks to household income risk. Econometrica 87 (1), 255–290.

[4] Bowman, D., D. Minehart, and M. Rabin (1999). Loss aversion in a consumption-savings model. Journal of Economic Behavior \& Organization 38 (2), 155–178.

[5] Browning, M., and Lusardi, A. (1996) Household saving: Micro theories and micro facts. Journal of Economic literature, 34(4), 1797-1855.

[6] Cagetti, Marco (2003) Wealth Accumulation Over the Life Cycle and Precautionary Savings. Journal of Business and Economic Statistics, 21(3), 339–353.

[7] Carroll, C. D. and Samwick, A. A. (1998) How important is precautionary saving? Review of Economics and Statistics, 80(3), 410-419.

[8] Carroll C.D., Kimball M.S. (2008) Precautionary Saving and Precautionary Wealth. In: Palgrave Macmillan (eds) The New Palgrave Dictionary of Economics. Palgrave Macmillan, London.

[9] Chapman, J., Snowberg, E., Wang, S., and Camerer, C. (2022). Looming Large or Seeming Small? Attitudes Towards Losses in a Representative Sample. CESifo Working Paper No. 9820.

[10] Christelis, D., D. Georgarakos, T. Jappelli, and M. van Rooij (2020). Consumption uncertainty and precautionary saving. The Review of Economics and Statistics 102 (1), 148–161.

[11] Deidda, M. (2013). Precautionary saving, financial risk, and portfolio choice. Review of Income and Wealth 59 (1), 133–156.

[12] Dhami, S. (2016). The foundations of behavioral economic analysis. Oxford University Press: Oxford.

[13] Dhami, S. (2019). The foundations of behavioral economic analysis: Behavioral Decision Theory. Volume 1. Oxford University Press: Oxford.

[14] Dhami, S. (2019). The foundations of behavioral economic analysis: Time discounting. Volume 3. Oxford University Press: Oxford.

[15] Dhami, S. (2020). The foundations of behavioral economic analysis: Bounded rationality. Volume 5. Oxford University Press: Oxford.

[16] Dhami, S., Hajimoladarvish, N., and Mamidi, P. (2022) Loss Aversion and Tax evasion: Theory and Evidence. Mimeo, University of Leicester.

[17] Drèze, J. and Modigliani, F. (1972) Consumption decisions under uncertainty. Journal of Economic Theory, 5, 308-35.

[18] Engen E. M., and Gruber, J. (2001) Unemployment insurance and precautionary saving. Journal of Monetary Economics, 47(3), 545-579.

[19] Gächter, S., Johnson, E. J., and Herrmann, A. (2022). Individual-level loss aversion in riskless and risky choices. Theory and Decision 92(3-4), 599-624.

[20] Giamattei, M., Yahosseini, K. S., Gächter, S. and Molleman, L. (2020). LIONESS Lab: a free web-based platform for conducting interactive experiments online. Journal of the Economic Science Association, 6, 95–111.

[21] Giles, J. and K. Yoo (2007). Precautionary behavior, migrant networks, and household consumption decisions: An empirical analysis using household panel data from rural China. The Review of Economics and Statistics 89 (3), 534–551.

[22] Greiner, B. (2015). Subject pool recruitment procedures: organizing experiments with ORSEE. Journal of the Economic Science Association, 1(1), 114-125.

[23] Gourinchas, Pierre-Olivier, and Jonathan A. Parker (2002) Consumption Over the Lifecycle. Econometrica, 70(1), 47–89.

[24] Guiso, L., Jappelli, T., and Terlizzese, D. (1992) Earnings uncertainty and precautionary saving. Journal of Monetary Economics, 30(2), 307–337.

[25] Ibanez, M., and Schneider, S. O. (2023) Income Risk, Precautionary Saving, and Loss Aversion – An Empirical Test. Discussion Papers of the Max Planck Institute for Research on Collective Goods 2023/6.

[26] Kahneman, D. and A. Tversky (1979). Prospect theory: An analysis of decision under risk. Econometrica 47 (2), 263–292.

[27] Kahneman, D., Tversky, A. (Eds.). (2000). Choices, values and frames. Cambridge: Cambridge University Press.

[28] Kimball, M. S. (1990) Precautionary Saving in the Small and in the Large. Econometrica: Journal of the Econometric Society, 53-73.

[29] Köszegi, B. and M. Rabin (2006). A Model of Reference-Dependent Preferences. The Quarterly Journal of Economics, 121(4): 1133–1165.

[30] Köszegi, B. and M. Rabin (2009). Reference-dependent consumption plans. American Economic Review 99 (3), 909–936.

[31] Leland, H. E. (1968) Saving and uncertainty: The precautionary demand for saving. The Quarterly Journal of Economics, 465-473.

[32] Lise, J. (2013). On-the-job search and precautionary savings. The Review of Economic Studies 80 (3), 1086–1113.

[33] Lugilde, A., R. Bande, and D. Riveiro (2019). Precautionary saving: A review of the empirical literature. Journal of Economic Surveys 33 (2), 481–515.

[34] Lusardi, A. (1998). On the importance of the precautionary saving motive. American Economic Review 88 (2), 449–453.

[35] Miller, B. L. (1976) The effect on optimal consumption of increased uncertainty in labor income in the multiperiod case. Journal of Economic Theory 13, 154-167.

[36] Novemsky, N., and Kahneman, D. (2005). The boundaries of loss aversion. Journal of Marketing Research. 42(2): 119–128.

[37] Pagel, M. (2017). Expectations-based reference-dependent life-cycle consumption. The Review of Economic Studies 84 (2), 885–934.

[38] Park, H. (2016). Loss aversion and consumption plans with stochastic reference points. The BE Journal of Theoretical Economics 16 (1), 303–336.

[39] Prelec, D., and Loewenstein, G. (1998). The Red and the Black: Mental accounting of savings and debt. Marketing Science 17(1): 4–28.

[40] Sandmo, A. (1970) The effect of uncertainty on saving decisions. The Review of Economic Studies, 353-360.

[41] Siegmann, A. (2002). Optimal saving rules for loss-averse agents under uncertainty. Economics Letters 77 (1), 27–34.

[42] Sibley, D.S. (1975) Permanent and transitory effects in a model of optimal consumption with wage income uncertainty. Journal of Economic Theory, 11, 68-82.

[43] Skinner, J. (1988) Risky income, life cycle consumption, and precautionary savings. Journal of Monetary Economics, 22(2), 237-255.

[44] Starmer, C. (2000). Developments in non-expected utility theory: the hunt for a descriptive theory of choice under risk. Journal of Economic Literature. 38(2): 332–382.

[45] Thaler, R. H. (2015). Misbehaving: The making of behavioral economics. W. W. Norton \& Company: New York.

[46] Thaler, R. (1985) Mental Accounting and Consumer Choice, Marketing Science, 4, 199-214. republished (2008) in Marketing Science

[47] Thaler, R. (1999) Mental accounting matters. Journal of Behavioral Decision Making 12(3): 183-206.

[48] Thaler, R. H. and S. Benartzi (2004). Save more tomorrow™: Using behavioral economics to increase employee saving. Journal of Political Economy 112 (S1), S164–S187.

[49] Zeldes, S. P. (1989) Optimal consumption with stochastic income: Deviations from certainty equivalence. The Quarterly Journal of Economics, 275-298.